AN EQUIVALENT CIRCUIT OF A MATRIX CONVERTER WITH PASSIVE OR ACTIVE LOADS

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Abstract: *In this paper, an equivalent circuit to obtain the steady-state performance of a system composed of matrix converter (MC) with its input filter feeding passive or active, an induction motor, loads is presented. The presented equivalent circuit is verified by comparing its results with previously published results. It is found that the presented equivalent circuit gives almost the same performance results obtained from the previously published results which proves the validity of this equivalent circuit.*
Key words: *matrix converter, LC-filter, Induction motor, Passive loads, Active loads, equivalent circuit.*

1. Introduction

The matrix converter is an AC/AC converter that gains a lot of attention by several authors because of its advantages. These advantages are its simple and compact power circuit because it does not require DC-link as in using inverters. It can provide output voltage with the desired magnitude and frequency, with its input and output currents having low harmonic content. It also can be operated at unity, leading and lagging input displacement factor regardless of the load type, and it has the ability of power regeneration [1-6].

The modeling and simulation of a system composed of a matrix converter feeding passive or active, an induction motor, loads was dealt with in [4].

In [4], the fundamental performance of such system was obtained by making harmonic analysis of the obtained instantaneous waveforms of the output whether voltages or currents which were obtained from numerical solution of the loads differential equation and the MC mathematical model.

In this paper, an equivalent circuit to directly obtain the fundamental steady-state performance of system composed of a matrix converter with an input LC-filter and feeding passive or active loads is derived. From this equivalent circuit, the input and output performance characteristics of the matrix converter/ passive or active loads systems can be

obtained directly without the need to use numerical methods of solution as given in [4]. The presented equivalent circuit is validated by comparing its results with the previously published results, which were obtained by numerical solution methods.

2. Method of analysis
2.1 Passive load case

For certain output frequency, voltage gain ratio of the MC, input displacement factor, load parameters and filter parameters, the fundamental frequency performance of the system under consideration, Fig. (1), in which the load in this case is the inductive passive load type, can be obtained by a method based on the process of referring the load parameters to the MC input side. During this process the principle of power invariance is taken into consideration assuming that the switching elements of the matrix converter are ideal. Thus, the system of the MC and its load is seen by its input as an equivalent impedance.

Therefore, the per-phase steady-state equivalent circuit of the system under consideration becomes as shown in Fig. (2). Where in this equivalent circuit V_s is the rms value of the AC supply voltage, I_s is the fundamental rms value of the supply current, V_{mci} is the fundamental MC input voltage, I_{mci} is the fundamental MC input current, X_{Lf} is the inductive reactance of the filter inductor, X_{cf} is the capacitive reactance of the filter capacitor.
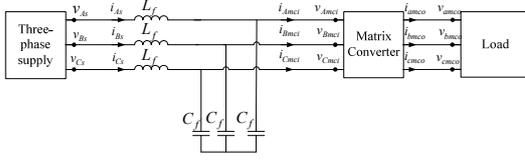
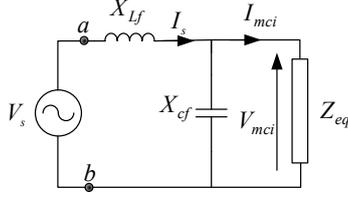**Fig. (1)** System under consideration at different loads.

Fig. (2) Steady-state equivalent circuit of matrix converter with its input filter feeding passive R-L load.

The equivalent impedance, Z_{eq}, can be obtained in the following manner.

When the harmonics are neglected, and the MC is assumed to be ideal, the input and output active power of the matrix converter are equal, thus:

$$V_{mci} I_{mci} \cos\varphi_1 = V_{mco} I_{mco} \cos\varphi_L \qquad (1)$$

where V_{mco} and I_{mco} are the fundamental-frequency components of the MC output voltage and output current respectively, $\cos\varphi_1$ is the input displacement factor of the matrix converter and $\cos\varphi_L$ is the load power factor at fundamental frequency.

The fundamental output voltage is related to the input voltage by the relationship:

$$V_{mco} = q V_{mci} \qquad (2)$$

where q is the voltage gain (or voltage transfer ratio) of the matrix converter.

Substituting, eqn. (2) into eqn. (1) and solving for the MC output current gives:

$$I_{mco} = I_{mci} \cos\varphi_1 /(q \cos\varphi_L) \qquad (3)$$

Eqn.(1) can be rewritten as:

$$I_{mci}^2 R_e = I_{mco}^2 R_L \qquad (4)$$

where R_e is the real part of the equivalent impedance of the matrix converter and its load, which is the equivalent resistance, and R_L is the load resistance.

Substituting eqn. (3) into eqn. (4) and solving for the equivalent resistance (R_e) the following expression is obtained:

$$R_e = R_L (\cos\varphi_1)^2 /(q \cos\varphi_L)^2 \qquad (5)$$

Thus:

$$R_e = [R_L^2 + (2\pi f_o L_L)^2](\cos\varphi_1)^2 /(q^2 R_L) \quad (6)$$

where L_L is the load inductance and f_o is the output fundamental frequency of the matrix converter. Using the obtained equivalent resistance, R_e, and the input displacement factor, $\cos\varphi_1$, the equivalent input impedance, Z_{eq}, of the MC with its load can be obtained from:

$$Z_{eq} = R_e / \cos\varphi_1 \qquad (7)$$

Thus the equivalent reactance, X_e, imaginary part of the equivalent impedance, of the MC with its load at the input of the MC can be obtained from:

$$X_e = \sqrt{Z_{eq}^2 - R_e^2} \qquad (8)$$

Therefore the equivalent impedance of the MC with its load can be defined as:

$$Z_{eq} = R_e \pm j X_e \qquad (9)$$

Where the negative sign is used when the input displacement factor is leading , while the positive sign is used when the input displacement factor is lagging.

The supply input current, I_s, can be obtained using the equivalent circuit, Fig. (2), from:

$$I_s = V_s / Z_{ab} \qquad (10)$$

where $Z_{ab} = jX_{Lf} + [-jX_{cf} Z_{eq}/(Z_{eq} - jX_{cf})]$

Thus, the matrix converter input voltage can be obtained from:

$$V_{mci} = V_s - I_s (jX_{Lf}) \qquad (11)$$

Consequently, the MC input current (I_{mci}) is obtained from the following relationship:

$$I_{mci} = V_{mci} / Z_{eq} \qquad (12)$$

and the output current of the matrix converter (I_{mco}) is obtained from eqn. (3).

When the input displacement factor, $\cos\varphi_1$, is unity the system of the MC with its load will be seen at its input as an equivalent resistance, as it is evident from eqn.(7). This equivalent resistance can be obtained from eqn.(6), by substitution with unity input displacement factor, i.e. $\cos\varphi_1 = 1$. Thus R_e is obtained as:

$$R_e = [R_L^2 + (2\pi 2_o L_L)^2]/(q^2 R_L) \qquad (13)$$

When the input filter is formed by a filter capacitor only, and the input displacement factor is unity, the steady-state equivalent circuit of the system shown in Fig. (2) will be reduced to that shown in Fig. (3); where the system of the MC with its load will be seen at its input as an equivalent resistance.
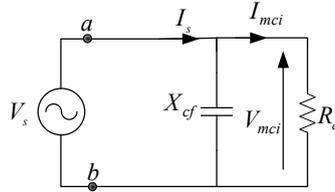

Fig. (3) Reduced steady-state equivalent circuit of the system under consideration.

In this case, the MC input voltage will be equal to the AC supply voltage and the MC input current can be obtained from:

$$I_{mci} = V_s / R_e \qquad (14)$$

and eqn. (3), which was used to obtain the MC output current, becomes:

$$I_{mco} = I_{mci} /(q\cos\varphi_L) \qquad (15)$$

The fundamental output voltage, V_{mco}, is obtained from eqn.(2).

2.2 Active load case

Fig. (1) shows the system under consideration in which the load in this case is active load (induction motor). The induction motor can be considered as an equivalent input impedance, Z_{inM}, at its input, i.e. at the output terminals of the matrix converter. This input impedance can be referred to the input terminals of the matrix converter in similar manner

as that described for the case of inductive passive load presented in Section (2.1).

Therefore, in order to present an equivalent circuit of the matrix converter feeding an induction motor with an input filter, the equivalent impedance, Z_{eqM}, is obtained in the following manner.

Assuming the induction motor is operated at certain condition which corresponds to certain motor slip, s_M, the induction motor input impedance can thus be obtained from its per-phase equivalent circuit shown in Fig. (4). Where in this equivalent circuit R_{sM} is the stator resistance, X_{sM} is the stator leakage reactance, R_{rM} is the rotor resistance, X_{rM} is the rotor leakage reactance and X_{mM} is the magnetizing reactance of the induction motor.

The induction motor input impedance is given by:

$$Z_{inM} = R_{sM} + jX_{sM} + Z_{mM} Z_{rM}/(Z_{mM} + Z_{rM}) \qquad (16)$$

where

$$Z_{mM} = jX_{mM} \ , \ Z_{rM} = R_{rM}/s_M + jX_{rM}$$

The real part of eqn. (16) represents the induction motor input resistance (R_{LM}), is given as:

$$R_{LM} = R_{sM} + \frac{X_{mM}^2 R_{rM}}{s_M[(R_{rM}/s_M)^2 + (X_{mM} + X_{rM})^2]}$$
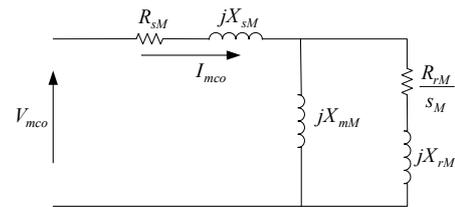
$$(17)$$

Fig. (4) Per-phase equivalent circuit of an induction motor.

The imaginary part of eqn. (16) represents the induction motor input reactance (X_{LM}), and is obtained as:

$$X_{LM} = X_{sM} + \frac{X_{mM}[(R_{rM}/s_M)^2 + X_{rM}(X_{mM} + X_{rM})]}{[(R_{rM}/s_M)^2 + (X_{mM} + X_{rM})^2]}$$

$$(18)$$

The steady-state equivalent circuit of the system under consideration becomes similar to that shown previously in Fig. (2) as shown in Fig. (5) in which the equivalent system impedance (Z_{eqM}) is obtained as:

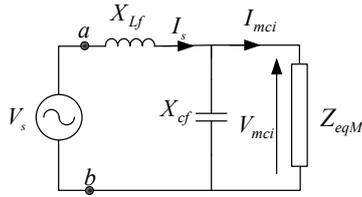$$Z_{eqM} = R_{eM} \pm jX_{eM} \qquad (19)$$

Fig. (5) Steady-state equivalent circuit of matrix converter with its input filter feeding active (induction motor) load.

Applying the same approach used for the passive R-L load the following expressions for the case of the induction motor are obtained.
The real part of the equivalent impedance of the matrix converter and the induction motor becomes:

$$R_{eM} = [R_{LM}^2 + (2\pi f_o L_{LM})^2]/(q^2 R_{LM}) \qquad (20)$$

where: $L_{LM} = X_{LM}/2\pi f_o$

The equivalent impedance in this case will become:

$$Z_{eqM} = R_{eM}/\cos\varphi_1 \qquad (21)$$

and the equivalent reactance of the system becomes:

$$X_{eM} = \sqrt{Z_{eqM}^2 - R_{eM}^2} \qquad (22)$$

The supply input current to the matrix converter feeding an induction motor becomes:

$$I_s = V_s/Z_{abM} \qquad (23)$$

where: $Z_{abM} = jX_{Lf} + [-jX_{cf}Z_{eqM}/(Z_{eqM} - jX_{cf})]$

The input current to the matrix converter becomes:

$$I_{mci} = V_{mci}/Z_{eqM} \qquad (24)$$

It is worth noting that, in this case when the matrix converter is feeding an induction motor load, and the input filter is formed by a filter capacitor

only with an input displacement factor of unity, the steady-state equivalent circuit of the system is shown in Fig. (3) as discussed in Section (2.1), with R_e replaced by R_{eM}.

3. Results
3.1 Passive, R-L, load results

The fundamental-frequency equivalent circuit for the matrix converter when feeding passive inductive load is validated by comparing the fundamental-frequency matrix converter output voltage, V_{mco}, and its output current, I_{mco}, obtained from previously published results [4], which were obtained using a numerical solution method, with those obtained from the proposed equivalent circuit of the MC / passive inductive load system when fed from a conventional AC supply. The results were obtained in Table (1) at the following conditions [4]:

- Unity input displacement factor
- Voltage gain ratio, q=0.4
- Output frequency, f_o=100 Hz
- Switching frequency, f_s=5 kHz
- Peak value of the input voltage, V_s= 230 V
- Input frequency, f_i =50 Hz
- Load resistance, R_L=10 Ω
- Load inductance, L_L=20 mH

Table (1) Comparison between results obtained from [4] and the proposed steady-state equivalent circuit approach for passive R-L load $q=0.4$, $f_o=100$ Hz, $V_s=230$V.

Item	Previously published results [4]	Proposed steady-state equivalent circuit results
V_{mco} , V	64	65
I_{mco} , A	4.06	4.05

Another set of results were obtained in Table (2) at the following conditions [4]:

- Unity input displacement factor
- Voltage gain ratio, q=0.85
- Output frequency, f_o=30 Hz
- Switching frequency, f_s=5 kHz
- Peak value of the input voltage, V_s= 110 V
- Input frequency, f_i =50 Hz
- Load resistance, R_L=10 Ω
- Load inductance, L_L=20 mH

Table (2) Comparison between results obtained from [4] and the proposed steady- state equivalent circuit approach for passive R-L load *q=0.85, f_o=30 Hz,V_s=110V.*

Item	Previously published results [4]	Proposed steady-state equivalent circuit results
V_{mco} , V	66.14	66.12
I_{mco} , A	6.18	6.19

Table (1) and Table (2) summarize the obtained results of the output voltage and output current of the MC, obtained from [4], when the load is a passive R-L load, and those obtained from the proposed fundamental- frequency equivalent circuit of the MC / inductive load system.

It is evident from Table (1) and Table (2) that the proposed method using the fundamental- frequency equivalent circuit approach gives almost identical results with the previously published results [4], which were obtained using numerical solution methods. This proves the validity of the proposed equivalent circuit presented.

3.2 Active, induction motor, load results

The fundamental-frequency equivalent circuit for the matrix converter when feeding an induction motor load (active load) whose data are given in appendix, is validated by comparing the fundamental-frequency MC output voltage, V_{mco}, and output current, I_{mco}, obtained from [4] with those obtained from the proposed equivalent circuit of the MC / induction motor load system when fed from a conventional AC supply. The results are shown in Table (3) at the following conditions [4]:

- Unity input displacement factor
- voltage gain ration, *q=0.35*
- peak value of the input voltage, *V_{im}=327 V*
- output frequency, *f_o=42 Hz*
- switching frequency, *f_s=5 kHz*
- Input frequency, *f_i* =60 Hz

In this case, the induction motor is operated at no load and with its speed adjusted at rated speed.

Table (3) Comparison between results obtained from [4] and the proposed steady- state equivalent circuit approach for induction motor load *q=0.35, f_o=42 Hz,V_s=327V.*

Item	Previously published results [4]	Proposed steady-state equivalent circuit results
V_{mco} , V	81.52	80.93
I_{mco} , A	5.57	5.56

Another set of results, for the same induction motor at the same loading and speed conditions as in the previous case, are given in Table (4) at the following conditions [4]:

- Unity input displacement factor.
- Voltage gain ratio, q=0.8
- Output frequency, f_o=70 Hz
- Switching frequency, f_s=5 kHz
- Peak value of the input voltage, V_s= 204 V
- Input frequency, f_i =60 Hz

Table (4) Comparison between results obtained from [4] and the proposed steady- state equivalent circuit approach for induction motor load *q=0.8, f_o=70 Hz,V_s=204V.*

Item	Previously published results [4]	Proposed steady-state equivalent circuit results
V_{mco} , V	115.32	115.4
I_{mco} , A	4.76	4.76

Table (3) and Table (4) show the obtained results of the output voltage and output current of the MC when feeding an induction motor, given in [4] and the results obtained from the proposed fundamental-frequency equivalent circuit of the MC / induction motor load system.

Table (3) and Table (4) show that the proposed approach using the fundamental- frequency equivalent circuit gives almost identical results with the previously published results obtained from [4] for the induction motor case, which were obtained numerically. This proves the validity of the proposed equivalent circuit for the case of a matrix converter feeding an induction motor.

4. Conclusion

An equivalent circuit for a matrix converter with passive or active load systems is derived. The presented equivalent circuit is verified by comparing its results with previously, numerically obtained, published results. The comparison showed that the compared results are almost identical which proves the validity of the presented equivalent circuit.

5. References

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6. Appendix

Three-phase induction motor, star-connected, 5 hp, 200 V, 60 Hz, 4-pole induction machine with the following parameters [4]:
R_s=0.277 Ω, L_s=0.0553 H, R_r=0.183 Ω, L_r=0.05606 H, L_m=0.0538 H, J=0.01667 kg/m^2 and rated speed=1767 rpm.